This is the October 23, 2005 version of the file `priusenergy.tex` by Chris Jacobsen which discusses energy conversion and storage in the 2004 model year Toyota Prius automobile.

1 Energy content of fuels

The energy content[1] of auto gasoline is

Fuel	**BTU/barrel**	**Joules/liter**	**kWh/gallon**
Conventional motor gasoline	5.25×10^6	3.49×10^7	36.7
Reformulated (oxygenated) gasoline	5.150×10^6	3.42×10^7	36.0

Any engine that uses heat released by burning fuel to do work is limited by the second law of thermodynamics to an efficiency no better than the Carnot limit of $1 - T_{\text{cold}}/T_{\text{hot}}$ with both temperatures measured on an absolute scale (such as °K). Ideally a gas engine could work at an efficiency of about $1 - (300°\text{K})(530°\text{K}) = 0.43$ or 43% efficiency with an ignition temperature of about 530°K[2], but typical real-life efficiencies are more like 30%.

2 Battery capacity of the Toyota Prius

According to a Toyota Europe press kit, the 2004 Prius battery capacity is 6.5 amp-hours at 201.6 Volts, or $(6.5 \cdot 201.6/1000) = 1.31$ kiloWatt-hours or kWh. Since a gallon of reformulated gasoline can release 36.0 kWh of heat energy (see Sec. 1), this would imply that the battery (with a mass of 39 kg) is equivalent to $(1.31/36.0) \simeq 1/27$ of a gallon of gas which would have a mass of

$$(1.31/36.0 \text{ US gallons}) \cdot \frac{3.7854 \text{ liters}}{1 \text{ US gallon}} \cdot \frac{0.73 \text{ kg}}{1 \text{ liter gasoline}} = 0.10 \text{ kg}$$

which shows why gasoline is much nicer to carry around than a battery as a light energy source. However, it's probably realistic to assume that only about 35% of the heat energy of gasoline is converted to work, while about 95% of the energy stored in the battery can be converted to work. This would mean that the work equivalent of the battery is more like $[(1.31 \cdot 0.95)/(36.0 \cdot 0.35)] \simeq 1/10$ of a gallon of gas which would have a mass of 0.27 kg.

3 Solar charging of the Prius battery?

The average solar energy received by a horizontal surface in the US ranges from about 3 to 6 kWh per square meter per day[3], though some states like Arizona and California can average more like

[1] U.S. Department of Energy, Energy Information Administration, Annual Energy Review 2004 [DOE/EIA-0384(2004)], Appendix A: Thermal Conversion Factors, `http://www.eia.doe.gov/emeu/aer/pdf/pages/sec13_1.pdf`.

[2] See *e.g.*, `http://hypertextbook.com/facts/2003/ShaniChristopher.shtml`

[3] See *e.g.*, `http://rredc.nrel.gov/solar/old_data/nsrdb/redbook/atlas/`.

10–14 kWh/m^2/day in the summer. Still, we will use 4.5 kWh/m^2/day as an average for the country. The area of the Prius' roof (not including the hatchback) is about 1.6 m^2, and the horizontal area of the hood is about 0.6 m^2. Also, typical solar panels have an efficiency of about 15%. As a result, if the Prius were equipped with 15% efficient solar panels on the roof and hood, it could collect up to about

$$\frac{4.5\text{ kWh}}{\text{m}^2 \cdot \text{day}} \cdot (2.2\text{ m}^2) \cdot (0.15\text{ efficiency}) = 1.5\text{ kWh/day}$$

of energy[4]. Again assuming 95% efficiency on converting this electrical energy to work and 35% efficiency on converting gasoline heat energy to work, this comes out to the equivalent of about $[(1.5 \cdot 0.95)/(36.0 \cdot 0.35)] = 0.11$ gallons or about 1/9 of a gallon of gasoline per day that one could avoid burning in such a solar-cell-equipped car. Summertime in the southwest US could give double or triple that number.

4 Regenerative braking in the Toyota Prius

The Prius shows you one green car (with the apparent texture of a grean leaf) for every 50 Wh=0.050 kWh it has recovered by using its regenerative braking system. Let's say that this recovered energy is converted to work at 95% efficiency, and that the heat energy of gasoline is converted to work at 35% efficiency. This tells us that one "green car" symbol is equivalent to $[(0.050 \cdot 0.95)/(36.0 \cdot 0.35)] = 0.0038$ gallons of gas utilized. This turns out to be equal to the work you could recover from burning 1.0 tablespoon of gasoline.

The kinetic energy of a Prius traveling at 60 miles per hour (curb weight 2890 pounds, plus 300 pounds cargo) is

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}(3190\text{ pounds})(60\frac{\text{miles}}{\text{hour}})^2 \cdot \frac{1\text{ kg}}{2.205\text{ pound}} \cdot (\frac{5280 \cdot 12\text{ inches}}{\text{mile}})^2 \\ &\quad \cdot (\frac{.0254\text{ meters}}{\text{inch}})^2 \cdot (\frac{\text{hour}}{3600\text{ seconds}})^2 \\ &= 520,400\text{ Joules} \cdot \frac{1\text{ Watt} \cdot \text{second}}{\text{Joule}} \cdot \frac{\text{hour}}{3600\text{ second}} = 145\text{ Wh}\end{aligned}$$

The Prius regenerative braking system is far from 100% efficient in recovering this energy; otherwise you would see three 50 Wh "green car" symbols every time you stopped from 60 miles per hour.

A Energy conversion factors

Here are some energy conversion factors. Go from millions of British thermal units (BTUs) per barrel to Joules/liter:

$$\frac{1\text{ BTU}}{\text{barrel}} \cdot \frac{1055.9\text{ Joule}}{\text{BTU}} \cdot \frac{1\text{ barrel}}{42\text{ US gallons}} \cdot \frac{1\text{ US gallon}}{3.7854\text{ liters}} = \frac{6.64\text{ Joule}}{\text{liter}}$$

[4]In Sec. 2 we found that the battery capacity is about 1.31 kWh, so the $\sim$ 1.5 kWh of solar energy one could get each day means it would take most of a day to fully charge a depleted battery.

Go from Joules/liter to kiloWatt-hours (kWh) per US gallon:

$$\frac{1\text{ Joule}}{\text{liter}} \cdot \frac{1\text{ Watt} \cdot \text{second}}{1\text{ Joule}} \cdot \frac{1\text{ kiloWatt}}{1000\text{ Watt}} \cdot \frac{1\text{ hour}}{3600\text{ second}} \cdot \frac{3.7854\text{ liters}}{\text{US gallon}} = \frac{1.052 \times 10^{-6}\text{ kWh}}{\text{gallon}}$$